Pergamon

Int. J. Heat Mass Transfer. Vol. 39, No. 13, pp. 2843–2845, 1996

0017-9310/96 $15.00+0.00

0017-9310(95)00361-4

TECHNICAL NOTES

Predicting the average heat transfer coefficient for an isothermal vertical circular disk with assisting and opposing combined forced and natural convection

G. L. WEDEKIND and C. J. KOBUS

School of Engineering and Computer Science, Oakland University, Rochester, MI 48309, U.S.A.

(*Received* 15 *May* 1995 *and in final form* 19 *September* 1995)

INTRODUCTION

Circular disks are an important geometry when considering electronic component cooling, such as the cooling of disk-type resistors and power transistors, or other related applications, such as the use of commercially available disk-type thermistors [1] for temperature and air flow measurements. Empirical correlations exist in the literature [1] for combined forced and natural convection from vertical circular disks for assisting flows, but not opposing flows. Also, to the best knowledge of the authors, there is no theoretical model currently available for predicting the convective heat transfer coefficient for assisting and opposing flows. Therefore, a conversion scheme is developed in the present research that utilizes a previously developed theoretical model [2], which was successful in predicting the average convective heat transfer coefficient for vertical flat plates experiencing combined forced and natural convection, to predict heat transfer coefficients for circular disks. The conversion scheme involves the concept of an effective flat plate length for a circular disk. The following flat plate solutions [2] may be used to predict the average heat transfer coefficient for circular disks by using this effective length, L^*:

Assisting flow

$$\frac{Nu_L}{Re_L^{1/2}} = \frac{2\lambda_b^{1/2} Pr}{3\sqrt{3}}\left(\frac{\lambda_a}{\lambda_b^2}\right)^{-1/2} Ri_L^{-1/2}\left\{\frac{1}{2}\left[1+\frac{24}{Pr}\left(\frac{\lambda_a}{\lambda_b^2}\right)Ri_L\right]^{1/2}+1\right\} \times\left\{\left[1+\frac{24}{Pr}\left(\frac{\lambda_a}{\lambda_b^2}\right)Ri_L\right]^{1/2}-1\right\}^{1/2}. \quad (1)$$

Opposing flow; natural convection initially dominates

$$\frac{Nu_L}{Re_L^{1/2}} = \frac{2\lambda_b^{1/2} Pr}{3\sqrt{3}}\left(\frac{\lambda_a}{\lambda_b^2}\right)^{-1/2} Ri_L^{-1/2}\left\{\left\{\frac{1}{2}\left[1+\frac{24}{Pr}\left(\frac{\lambda_a}{\lambda_b^2}\right)Ri_L\right]^{1/2}-1\right\} \times\left\{1+\left[1+\frac{24}{Pr}\left(\frac{\lambda_a}{\lambda_b^2}\right)Ri_L\right]^{1/2}\right\}^{1/2}+\frac{\sqrt{2}}{2}\right\}. \quad (2)$$

Opposing flow; forced convection initially dominates

$$\frac{Nu_L}{Re_L^{1/2}} = \frac{2\lambda_b^{1/2} Pr}{3\sqrt{3}}\left(\frac{\lambda_a}{\lambda_b^2}\right)^{-1/2} Ri_L^{-1/2}\left\{\frac{1}{2}\left[1-\frac{24}{Pr}\left(\frac{\lambda_a}{\lambda_b^2}\right)Ri_L\right]^{1/2}+1\right\} \times\left\{1-\left[1-\frac{24}{Pr}\left(\frac{\lambda_a}{\lambda_b^2}\right)Ri_L\right]^{1/2}\right\}^{1/2}. \quad (3)$$

The coefficients, λ_a and λ_b, are constants and functions only of the Prandtl number [2]. It should be noted that virtually all the differences between the three different flow configurations for the average heat transfer characteristics manifest themselves in sign changes. Also, the negative terms within the radicals of equation (3) restrict the range of applicability of this particular solution in terms of the Richardson number. In prior research [1], assisting flow experiments were performed to measure the average heat transfer coefficient with air for several circular disk heat transfer models of various diameters. The above experimentation was expanded to include opposing flow as well as assisting flow. The experimentation was identical to that done by Kobus and Wedekind [2], except that the experimental heat transfer models were circular disks rather than flat plates. Therefore, a description of the experimentation will not be repeated here.

CONVERTED EXPERIMENTAL DISK DATA

It was recognized by the authors, because of previous investigations [1–3], that experimental heat transfer data for thin circular disks was very similar to that of flat plates. It was postulated that the data for both geometric configurations might be collapsed,† albeit for a limited range of the governing parameters. The concept of an equivalent flat plate with an effective length for the disk was developed where the effective length was formulated as the average distance that a fluid particle would travel in the disk boundary layer, the local 'length' of the disk varying between zero at the two sides, to a maximum equal to the disk diameter, d, at the centerline. Utilizing the integral form of the mean value theorem, the effective length of the disk can be modeled as

$$L^* = \frac{1}{\pi}\int_{\theta=0}^{\pi} d\sin\theta\,\mathrm{d}\theta = \frac{2d}{\pi}. \quad (4)$$

Hence, utilizing equation (4) and the experimental disk data for combined forced and natural convective heat transfer available in the literature [1], the dimensionless par-

† Some investigators have striven, with limited success, to obtain a characteristic length such that experimental data for a variety of geometric configurations could be collapsed to a single curve. The interested reader should refer to the works of Sparrow and Stretton [4], Sparrow and Ansari [5], Lienhard [6] and Hassani and Hollands [7].

NOMENCLATURE

d	disk diameter [m]	L^*	effective length [m]
Gr_L	Grashof number, $\rho_g^2\beta(T_w - T_f)L^3/\mu^2$	Nu_L	Nusselt number, hL/k
h	convective heat transfer coefficient [W m^{-2} °C^{-1}]	Pr	Prandtl number, $\mu c_p/k$
		Re_L	Reynolds number, $\rho u_f L/\mu$
L	total plate length [m]	Ri_L	Richardson number, Gr_L/Re_L^2.

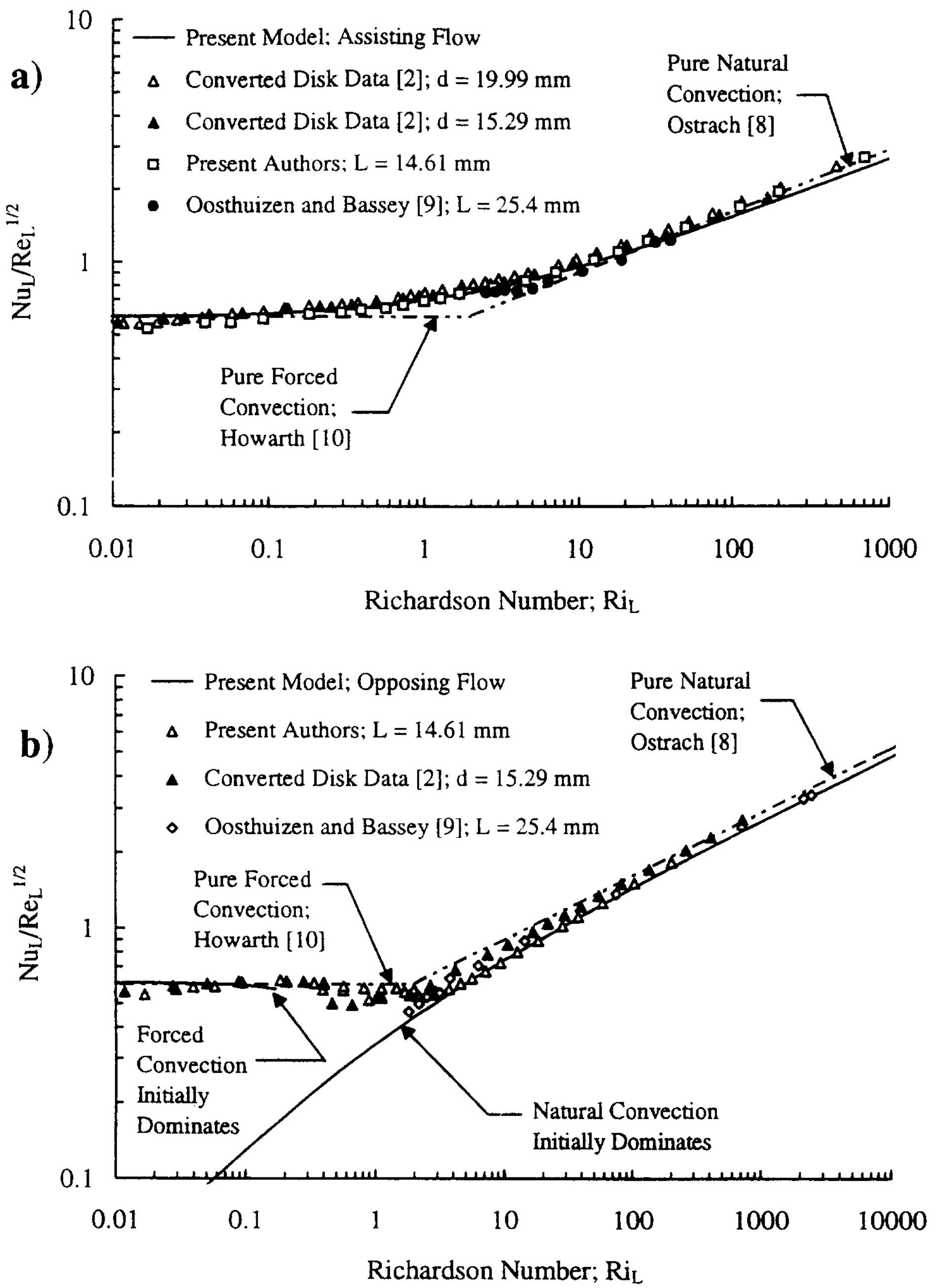

Fig. 1. Average combined forced and natural convective heat transfer characteristics from an isothermal vertical effective flat plate in air; converted disk data; (a) assisting flow, (b) opposing flow.

ameters may be converted from the disk to an equivalent plate with an effective length, L^*. However, the conversion was seen to be valid only for a limited range of the Grashof number based of the effective length. That is, based on the empirical correlation for pure natural convection from a thin circular disk [1], after a conversion of the dimensionless parameters to an effective length, utilizing equation (4), a comparison was made with the classic flat plate average pure natural convection solution [8]. A range restriction was established by comparison, where the converted empirical correlation for disks [1] was within 15% of the flat plate solution [8]. The result was that the disk data may be converted quite accurately for a restricted range of the Grashof number, $4000 \leqslant Gr_{L^*} \leqslant 50\,000$, based on the effective length, L^*, and specified for air ($Pr = 0.72$). The experimental data for two of the heat transfer models presented by Kobus and Wedekind [1] met the criteria, where the disk diameters, $d = 15.29$ and 19.99 mm, corresponded to $Gr_{L^*} = 7400$ and $17\,500$, respectively. Since the converted disk correlation has a slightly lower slope than the flat plate solution, any converted experimental disk data falling short of the specified criteria would appear high, while experimental data that was beyond the criteria would appear low.

Experimental data: assisting flow

The present model is compared with the experimental converted results of the prior research [1], for assisting forced and natural convective heat transfer, and is depicted in dimensionless form in Fig. 1(a) along with one of the experimental flat plate data sets, each from the present authors and of Oosthuizen and Bassey [9]. Referring to Fig. 1(a), excellent agreement is seen between the assisting flow solution of the present model, equation (1), the converted experimental disk data [1], and the experimental flat plate data of the authors [2] and of Oosthuizen and Bassey [9]. All of the experimental flat plate data of the authors and of Oosthuizen and Bassey were not presented to prevent data clutter, but it is clear that it would have fallen directly on top of the experimental data that was presented in Fig. 1(a). Thus, it is apparent that the present model will predict the heat transfer characteristics of disks as well as flat plates, albeit for an abbreviated range of the Grashof number.

Experimental data: opposing flow

A comparison of the two opposing flow solutions of the present model and the converted experimental results, for opposing forced and natural convective heat transfer, is depicted in dimensionless form in Fig. 1(b), along with a single set of experimental flat plate data of the authors [2] and of Oosthuizen and Bassey [9]. Again, referring to Fig. 1(b), excellent agreement is seen between the present model, the experimental disk data, converted to an effective length by equation (4), and the experimental flat plate data of the authors [2] and of Oosthuizen and Bassey [9]. As with assisting flow, the whole of the experimental flat plate data of the authors and of Oosthuizen and Bassey were not presented to prevent data clutter, but it is again clear that it would have fallen directly on top of the data that were presented in Fig. 1(b). It should be noted that there seems to be a 'gap' between the two opposing flow solutions, and that this represents the mathematical limitations of the model at this time.

CONCLUSIONS

A conversion scheme has been developed extending the predictive capability of a previously developed theoretical model for a flat plate [2] to circular disks, for both assisting and opposing flow situations. The conversion scheme is valid for an appreciable range of Grashof number. Also, although no experimental data currently exists, the solutions are capable of handling fluids for Prandtl numbers other than unity [2]. It should be pointed out that, although an empirical correlation does exist predicting the average heat transfer characteristics of circular disks in assisting flow [1], no such correlation, to the best knowledge of the authors, exists for opposing flow. Therefore, although the conversion of the experimental disk data to an effective length is valid only for a restricted range of the Grashof number, as discussed earlier, it is seen at this time to be the *only* way to predict heat transfer coefficients for circular disks in the case of opposing flow.

REFERENCES

1. C. J. Kobus and G. L. Wedekind, An experimental investigation into forced, natural and combined forced and natural convective heat transfer from stationary isothermal circular disks, *Int. J. Heat Mass Transfer* **38,** 3329–3339 (1995).
2. C. J. Kobus and G. L. Wedekind, Modeling the local and average heat transfer coefficient for an isothermal vertical flat plate with assisting and opposing combined force and natural convection, *Int. J. Heat Mass Transfer* **39,** 2723–2733 (1996).
3. G. L. Wedekind, Convective heat transfer measurement involving flow past stationary circular disks, *J. Heat Transfer* **111,** 1098–1100 (1989).
4. E. M. Sparrow and A. J. Stretton, Natural convection from variously oriented cubes and from other bodies of unity aspect ratio, *Int. J. Heat Mass Transfer* **28,** 741–752 (1985).
5. E. M. Sparrow and M. A. Ansari, A refutation of King's rule for multi-dimensional external convection, *Int. J. Heat Mass Transfer* **26,** 1357–1364 (1983).
6. J. H. Lienhard, On the commonality of equations for natural convection from immersed bodies, *Int. J. Heat Mass Transfer* **16,** 2121–2123 (1973).
7. A. V. Hassani and K. G. T. Hollands, A simplified method for estimating natural convection heat transfer from bodies of arbitrary shape, *Presented at the 24th National Heat Transfer Conference*, Pittsburgh, Pennsylvania, 9–12 August (1987)
8. S. Ostrach, An analysis of laminar free-convection and heat transfer about a flat plate parallel to the direction of the generating body force, NACA Report 1111 (1953).
9. P. H. Oosthuizen and M. Bassey, An experimental study of combined forced and free convection heat transfer from flat plates to air at low Reynolds numbers, *J. Heat Transfer* **95,** 120–121 (1973).
10. L. Howarth, On the solution of the laminar boundary layer equations, *Proc. R. Soc., Lond.* **164A,** 547–579 (1938).